PLASTICITY DURING VARIABLE LOADINGS IN ELECTROMAGNETIC FIELDS*

V. V. Kolokol'chikov,
V. V. Moskvitin, and
B. L. Sidorov

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The basic strain equations of cyclic loadings are presented in [1], and theorems on cyclic loadings of isotropic plastic materials are proved. Masing's principle [2] and the equations of Shneiderovich [3] are also used in isotropic plasticity. A phenomenological model of an elastic body, polarized and magnetized without hysteresis, taking account of gyromagnetic effects and the finite values of the strains, is constructed in [4]. Models of a continuous medium with electromagnetic moments which take account of the effects of magnetic hysteresis and plastic deformations within the framework of the theory of relativity are formulated in [5]. Models of magnetoelastic media which take account of magnetic hysteresis and plastic deformations are treated in [6] on the basis of a variational equation of the mechanics of continuous media [7]. In the present article we discuss the development of the deformation theory of cyclic loadings for anisotropic ferromagnetic and ferroelectric materials.

1. We consider a solid ferromagnetic or ferroelectric body of arbitrary shape having a volume V, and bounded by the surface S. In the undeformed coordinate system $x_i$ we introduce $u_{i(n)}$, the components of the n-th displacement vector and $\varepsilon_{ij(n)}$, the components of the strain tensor for the n-th loading, the components of the strain tensor for the n-th loading. All quantities for the n-th loading are labeled with the index n.

Henceforth we assume the following: The body is subjected to body forces with a volume density $F_{i(n)}$ and stresses $T_{i(n)}$ on the surface S. The magnetic and electric fields are determined from Maxwell's equations and the appropriate boundary conditions for problems of magnetostatics or electrostatics [8, 9], taking account of the deformation of the material.

For definiteness we consider a continuous ferromagnetic medium, since analogous results for a continuous ferroelectric medium can be obtained by replacing $H_{i(n)}$ by $E_{i(n)}$, the components of the magnetization vector $I_{i(n)}$ by the components of the polarization vector $P_{i(n)}$, and adding the body forces $\rho_{(n)} E_{i(n)}$.

The equations of equilibrium, taking account of ponderomotive forces, have the form

$$\frac{\partial \sigma_{ij(n)}}{\partial x_j} + F_i(I_{(n)}, H_{(n)}) + F_{i(n)} = 0,$$
$$F_i(I_{(n)}, H_{(n)}) = \frac{1}{2}\left(I_{k(n)}\frac{\partial H_{k(n)}}{\partial x_i} - H_{k(n)}\frac{\partial I_{k(n)}}{\partial x_i}\right) - \frac{1}{2}\operatorname{rot}(\mathbf{I}_{(n)}\times\mathbf{H}_{(n)}). \quad (1.1)$$

We take the expression for the total stress tensor in the form [9]

$$T_{ij(n)} = \sigma_{ij(n)} + \sigma_{ij}(B_{(n)}, H_{(n)}),$$
$$\sigma_{ij}(B_{(n)}, H_{(n)}) = (1/8\pi)(B_{i(n)} H_{j(n)} + B_{j(n)} H_{i(n)} - B_{k(n)} H_{k(n)} \delta_{ij}), \quad (1.2)$$

where the $B_{i(n)}$ are the components of the magnetic induction in Gaussian units for the n-th loading

$$B_{i(n)} = H_{i(n)} + 4\pi I_{i(n)}. \quad (1.3)$$

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The boundary conditions for stresses on surfaces with normals having direction cosines $n_i$ are written in the form [8]

$$T_{ij(n)}n_j = T_{i(n)} + T_i(H_{0(n)}),\ \ T_i(H_{0(n)}) = (\mu_0/8\pi)\left(2H_{0i(n)}H_{0j(n)} - H^2_{0(n)}\delta_{ij}\right)n_j, \tag{1.4}$$

where the $H_{0i(n)}$ are the components of the magnetic field outside the ferromagnet and $\mu_0$ is the magnetic permeability of a nonferromagnetic medium, assumed equal to unity.

Suppose for each n-th mechanical and magnetic loading these are stresses $\sigma_{ij(n)}$, strains $\varepsilon_{eij(n)}$, magnetic field $H_{i(n)}$, and a magnetization $I_{i(n)}$. We represent the stress deviators $S_{ij(n)}$ for the n-th loading

$$S_{ij(n)} = \sigma_{ij(n)} - (1/3)\sigma_{\alpha\alpha(n)}\delta_{ij} \tag{1.5}$$

as the sum of three terms, and the field $H_{i(n)}$ as the sum of two terms:

$$S_{ij(n)} = S_{ij}^{(n)} + S_{ij}^{[n]} + S_{ij}^{(In)},\quad H_{i(n)} = H_i^{(In)} + H_i^{(\varepsilon n)}, \tag{1.6}$$

where $S_{ij}^{(n)}$ is the potential mechanical part of the stress deviator, $S_{ij}^{[n]}$, nonpotential mechanical part of the stress deviator; $S_{ij}^{(In)}$, magnetostriction part of the stress deviator; $H_i^{(In)}$, magnetic part of the field; and $H_i^{(\varepsilon n)}$, mechanostriction part of the field.

For the first loading we take the following from the conditions for a potential and the three-dimensional linearity of the relations between $\sigma$, H $\sim$ $\varepsilon$, I:

$$\begin{gathered}\sigma_{ij(1)} = k_{ij\alpha\beta}\ (\varepsilon_{(1)}, I_{(1)})\varepsilon_{\alpha\beta(1)} + q_{\alpha\beta ij}(\varepsilon_{(1)}, I_{(1)})I_{\alpha(1)}I_{\beta(1)},\\ H_{i(1)} = \eta_{ij}(\varepsilon_{(1)}, I_{(1)})I_{j(1)} + 2q_{ij\alpha\beta}(\varepsilon_{(1)}, I_{(1)})I_{j(1)}\varepsilon_{\alpha\beta(1)},\\ k_{ii\alpha\beta} \equiv 9K_{ii\alpha\beta} = \text{const},\ k_{ij\alpha\beta} = k_{\alpha\beta ij} = k_{ji\alpha\beta} = k_{ij\beta\alpha},\\ q_{\alpha\beta ij} = q_{\beta\alpha ij} = q_{\alpha\beta ji},\ \eta_{ji} = \eta_{ij}.\end{gathered} \tag{1.7}$$

Equations (1.7) take account of the initial anisotropy. We introduce dimensionless constant tensors and label them with asterisk superscipts:

$$\begin{gathered}\mu^*_{ij\alpha\beta} = \frac{1}{2\mu}\left[k_{ij\alpha\beta}(0, 0) - \frac{1}{3}\delta_{ij}k_{\gamma\gamma\alpha\beta}(0, 0)\right],\\ \mu^{(I*)}_{ij\alpha\beta} = \left[q_{\alpha\beta ij}(0, 0) - \frac{1}{3}\delta_{ij}q_{\alpha\beta\gamma\gamma}(0, 0)\right],\ \ q^*_{\alpha\beta ij} = q_{\alpha\beta ij}(0, 0),\ \ \eta^*_{ij} = \eta_{ij}(0, 0),\end{gathered} \tag{1.8}$$

where $\mu$ is the arbitrary shear modulus. Here we have used the fact that $H_i$ and $I_i$ have the same dimensions.

We introduce the tensors $\mu^{(*)}_{ij\alpha\beta}$ and $\mu^{*}_{ij\alpha\beta}$ which result from the symmetrization and antisymmetrization of the dimensionless shear modulus tensor with respect to pairs of indices:

$$\mu^{(*)}_{ij\alpha\beta} = \frac{1}{2}(\mu^*_{ij\alpha\beta} + \mu^*_{\alpha\beta ij}),\quad \mu^{[*]}_{ij\alpha\beta} = \frac{1}{2}(\mu^*_{ij\alpha\beta} - \mu^*_{\alpha\beta ij}). \tag{1.9}$$

The tensor $\mu^{[*]}_{ij\alpha\beta}$ is not equal to zero for crystals of the monoclinic and triclinic systems. We introduce the quantities

$$\begin{gathered}e_{ij}^{(*n)} = \mu^{(*)}_{ij\alpha\beta}\varepsilon_{\alpha\beta(n)},\quad e_{ij}^{[*n]} = \mu^{[*]}_{ij\alpha\beta}\varepsilon_{\alpha\beta(n)},\\ e_{ij}^{(I*n)} = \frac{1}{2\mu}\mu^{(I*)}_{ij\alpha\beta}I_{\alpha(n)}I_{\beta(n)},\ \ I_i^{(*n)} = \overset{*}{\eta}_{i\alpha}I_{\alpha(n)},\ \ I_i^{(\varepsilon*n)} = \overset{*}{q}_{ij\alpha\beta}I_{j(n)}\varepsilon_{\alpha\beta(n)},\end{gathered} \tag{1.10}$$

which are, respectively, the reduced strain deviator taking account of anisotropy (the deviator if the strains satisfy the restriction $e^{[*]}_{ii} = 0$), reduced strain tensor taking account of the asymmetry of the shear modulus tensor with respect to pairs of indices, the reduced magnetostriction strain deviator, the reduced magnetization vector, and the reduced mechanostriction magnetization vector.

It is necessary to introduce increments of the quantities appearing in the material relations for the n-th and (n − 1)-th loadings:

$$\begin{gathered}\tilde{\sigma}^{(n)}=(1/3)(-1)^n(\sigma_{\alpha\alpha(n-1)}-\sigma_{\alpha\alpha(n)}),\quad \tilde{S}_{ij}^{(n)}=(-1)^n(S_{ij}^{(n-1)}-S_{ij}^{(n)}),\\ \tilde{S}_{ij}^{[n]}=(-1)^n(S_{ij}^{[n-1]}-S_{ij}^{[n]}),\quad \tilde{S}_{ij}^{(In)}=(-1)^n(S_{ij}^{(In-1)}-S_{ij}^{(In)}),\\ \tilde{H}_i^{(In)}=(-1)^n(H_i^{(In-1)}-H_i^{(In)}),\quad \tilde{H}_i^{(\varepsilon n)}=(-1)^n(H_i^{(\varepsilon n-1)}-H_i^{(\varepsilon n)}),\\ \tilde{\varepsilon}^{(*n)}=(1/K)(-1)^n K_{ii\alpha\beta}(\varepsilon_{\alpha\beta(n-1)}-\varepsilon_{\alpha\beta(n)}),\\ \tilde{e}_{ij}^{(*n)}=(-1)^n(e_{ij}^{(*n-1)}-e_{ij}^{(*n)}),\quad \tilde{e}_{ij}^{[*n]}=(-1)^n(e_{ij}^{[*n-1]}-e_{ij}^{[*n]}),\\ \tilde{e}_{ij}^{(I*n)}=(-1)^n(e_{ij}^{(I*n-1)}-e_{ij}^{(I*n)}),\quad \tilde{I}_i^{(*n)}=(-1)^n(I_i^{(*n-1)}-I_i^{(*n)}),\\ \tilde{I}_i^{(\varepsilon*n)}=(-1)^n(I_i^{(\varepsilon*n-1)}-I_i^{(\varepsilon*n)}),\end{gathered}\tag{1.11}$$

where K is the arbitrary bulk modulus. Following [10] we introduce for the tensors $\tilde{S}_{ij}^{(n)}$, $\tilde{S}_{ij}^{[n]}$, $\tilde{S}_{ij}^{(In)}$, $\tilde{e}_{ij}^{(*n)}$, $\tilde{e}_{ij}^{[*n]}$, $\tilde{e}_{ij}^{(I*n)}$, and the vectors $\tilde{H}_i^{(In)}$, $\tilde{H}_i^{(\varepsilon n)}$, $\tilde{I}_i^{(*n)}$ and $I_i^{(\varepsilon*n)}$ the direction tensors and vectors, denoting them by superscripts 1, by the formulas

$$\begin{gathered}\tilde{S}_{ij}^{(1n)}=\tilde{S}_{ij}^{(n)}/(\tilde{S}_{\alpha\beta}^{(n)}\tilde{S}_{\alpha\beta}^{(n)})^{1/2},\quad \tilde{S}_{ij}^{[1n]}=\tilde{S}_{ij}^{[n]}/(\tilde{S}_{\alpha\beta}^{[n]}\tilde{S}_{\alpha\beta}^{[n]})^{1/2},\\ \tilde{S}_{ij}^{(1In)}=\tilde{S}_{ij}^{(In)}/(\tilde{S}_{\alpha\beta}^{(In)}\tilde{S}_{\alpha\beta}^{(In)})^{1/2},\quad \tilde{e}_{ij}^{(1*n)}=\tilde{e}_{ij}^{(*n)}/(\tilde{e}_{\alpha\beta}^{(*n)}\tilde{e}_{\alpha\beta}^{(*n)})^{1/2},\\ \tilde{e}_{ij}^{[1*n]}=e_{ij}^{[*n]}/(\tilde{e}_{\alpha\beta}^{[*n]}\tilde{e}_{\alpha\beta}^{[*n]})^{1/2},\quad \tilde{e}_{ij}^{(1I*n)}=\tilde{e}_{ij}^{(I*n)}/(\tilde{e}_{\alpha\beta}^{(I*n)}\tilde{e}_{\alpha\beta}^{(I*n)})^{1/2};\end{gathered}\tag{1.12}$$

$$\begin{gathered}\tilde{H}_i^{(1In)}=\tilde{H}_i^{(In)}/(\tilde{H}_\alpha^{(In)}\tilde{H}_\alpha^{(In)})^{1/2},\quad \tilde{H}_i^{(1\varepsilon n)}=\tilde{H}_i^{(\varepsilon n)}/(\tilde{H}_\alpha^{(\varepsilon n)}\tilde{H}_\alpha^{(\varepsilon n)})^{1/2},\\ \tilde{I}_i^{(1*n)}=\tilde{I}_i^{(*n)}/(\tilde{I}_\alpha^{(*n)}\tilde{I}_\alpha^{(*n)})^{1/2},\quad \tilde{I}_i^{(1\varepsilon*n)}=\tilde{I}_i^{(\varepsilon*n)}/(\tilde{I}_\alpha^{(\varepsilon*n)}\tilde{I}_\alpha^{(\varepsilon*n)})^{1/2}.\end{gathered}$$

In [1] it is assumed that the direction stress and strain increment tensors coincide, generalizing to the case of variable loadings the idea of the coincidence of the stress and strain direction tensors in the theory of small elastic—plastic deformations [10]. We generalize the idea of [1] to the case of variable mechanical and magnetic loadings of anisotropic materials, assuming the equality of the direction stress increment tensors $\tilde{S}_{ij}^{(1n)}$, $\tilde{S}_{ij}^{[1n]}$, $\tilde{S}_{ij}^{(1In)}$ and the direction tensors $\tilde{e}_{ij}^{(1*n)}$, $\tilde{e}_{ij}^{[1*n]}$, $\tilde{e}_{ij}^{(1I*n)}$, respectively, and the quality of the direction vectors of the increments of the magnetic field $\tilde{H}_i^{(1n)}$, $\tilde{H}_i^{(1\varepsilon n)}$ and the direction vectors of the increments of the magnetization $\tilde{I}_i^{(1*n)}$ and $\tilde{I}_i^{(1\varepsilon*n)}$, respectively. Then for the n-th loading the relations between $\tilde{\sigma}_{ij(n)}$, $\tilde{H}_{i(n)}$ and $\tilde{\varepsilon}_{ij(n)}\tilde{I}_{i(n)}$ are determined by Eqs. (1.5) and (1.6), but written for increments of the quantities, labeled with a ~, and the equations:

$$\begin{gathered}\tilde{\sigma}^{(n)}=3K\tilde{\varepsilon}^{(*n)}+3\tilde{Q}^{(*n)},\quad \tilde{Q}^{(*n)}=\left(\frac{1}{9}\right)\tilde{g}^{(n)}q^{*}_{\alpha\beta ii}\tilde{I}_{\alpha(n)}\tilde{I}_{\beta(n)},\\ \tilde{S}_{ij}^{(n)}=\frac{2\tilde{\sigma}_{\mathrm{i}}^{(n)}}{3\tilde{\varepsilon}_{\mathrm{i}}^{(*n)}}\tilde{e}_{ij}^{(*n)},\quad \tilde{S}_{ij}^{[n]}=\frac{2\tilde{\sigma}_{\mathrm{i}}^{[n]}}{3\tilde{\varepsilon}_{\mathrm{i}}^{[*n]}}\tilde{e}_{ij}^{[*n]},\\ \tilde{S}_{ij}^{(In)}=\frac{2\tilde{\sigma}_{\mathrm{i}}^{(In)}}{3\tilde{\varepsilon}_{\mathrm{i}}^{(I*n)}}\tilde{e}_{ij}^{(I*n)},\quad \tilde{H}_i^{(n)}=\frac{\tilde{H}^{(n)}}{\tilde{I}^{(*n)}}\tilde{I}_i^{(*n)},\quad \tilde{H}_i^{(\varepsilon n)}=\frac{\tilde{H}^{(\varepsilon n)}}{\tilde{I}^{(\varepsilon*n)}}\tilde{I}_i^{(\varepsilon*n)},\end{gathered}\tag{1.13}$$

where

$$\begin{gathered}\tilde{\sigma}_{\mathrm{i}}^{(n)}=\left(\frac{3}{2}\tilde{S}_{ij}^{(n)}\tilde{S}_{ij}^{(n)}\right)^{1/2};\quad \tilde{\varepsilon}_{\mathrm{i}}^{(*n)}=\left(\frac{2}{3}\tilde{e}_{ij}^{(*n)}\tilde{e}_{ij}^{(*n)}\right)^{1/2};\\ \tilde{\sigma}_{\mathrm{i}}^{[n]}=\left(\frac{3}{2}\tilde{S}_{ij}^{[n]}\tilde{S}_{ij}^{[n]}\right)^{1/2};\quad \tilde{\varepsilon}_{\mathrm{i}}^{[*n]}=\left(\frac{2}{3}\tilde{e}_{ij}^{[*n]}\tilde{e}_{ij}^{[*n]}\right)^{1/2};\end{gathered}\tag{1.14}$$

$$\widetilde{\sigma}_{i}^{(In)} = \left(\frac{3}{2}\widetilde{S}_{ij}^{(In)}\widetilde{S}_{ij}^{(In)}\right)^{1/2}; \quad \widetilde{\varepsilon}_{i}^{(I*n)} = \left(\frac{2}{3}\widetilde{e}_{ij}^{(I*n)}\widetilde{e}_{ij}^{(I*n)}\right)^{1/2};$$
$$\widetilde{H}^{(n)} = (\widetilde{H}_i^{(n)}\widetilde{H}_i^{(n)})^{1/2}; \quad \widetilde{I}^{(*n)} = (\widetilde{I}_i^{(*n)}\widetilde{I}_i^{(*n)})^{1/2};$$
$$\widetilde{H}^{(\varepsilon n)} = (\widetilde{H}_i^{(\varepsilon n)}\widetilde{H}_i^{(\varepsilon n)})^{1/2}; \quad \widetilde{I}^{(\varepsilon * n)} = (\widetilde{I}_i^{(\varepsilon * n)}\widetilde{I}_i^{(\varepsilon * n)})^{1/2}.$$

We obtained the bulk properties in (1.13) by assuming the validity of the relation of bulk quantities for the base load, but written in terms of increments. Equations (1.13) are supplemented by the assumption of the existence of six universal relations among invariants, which can be written as a consequence of the hypothesis of the potential character of the material equations in increments in the form

$$\begin{gathered}
\widetilde{\sigma}_{i}^{(n)} = 3\widetilde{\varepsilon}_{i}^{(*n)}\widetilde{M}^{(n)}, \quad \widetilde{\sigma}_{i}^{[n]} = 3\widetilde{\varepsilon}_{i}^{[*n]}\mu, \quad \mu = \text{const}, \\
\widetilde{\sigma}_{i}^{(In)} = 3\varepsilon_{i}^{(I*n)}\mu\widetilde{q}^{(n)}, \quad \widetilde{H}^{(In)} = \widetilde{I}^{(*n)}\widetilde{\eta}^{(n)}, \\
\widetilde{H}^{(\varepsilon n)} = 2\widetilde{I}^{(\varepsilon * n)}\widetilde{q}^{(n)}, \quad \widetilde{Q}^{(*n)} = \left(\frac{1}{9}\right) q^{*}_{\alpha\beta\gamma\gamma}\widetilde{I}_{\alpha(n)}\widetilde{I}_{\beta(n)}\widetilde{q}^{(n)}, \\
\widetilde{M}^{(n)} = \widetilde{M}^{(n)}\left(\widetilde{e}_{ij}^{(*n)}\widetilde{\varepsilon}_{ij(n)}, \quad \widetilde{I}_{i}^{(*n)}\widetilde{I}_{i(n)}, \quad \widetilde{I}_{i}^{(\varepsilon * n)}\widetilde{I}_{i(n)}\right), \\
\widetilde{q}^{(n)} = \widetilde{q}^{(n)}\left(\widetilde{e}_{ij}^{(*n)}\widetilde{\varepsilon}_{ij(n)}, \quad \widetilde{I}_{i}^{(*n)}\widetilde{I}_{i(n)}, \quad \widetilde{I}_{i}^{(\varepsilon * n)}\widetilde{I}_{i(n)}\right), \\
\widetilde{\eta}^{(n)} = \widetilde{\eta}^{(n)}\left(\widetilde{e}_{ij}^{(*n)}\widetilde{\varepsilon}_{ij(n)}, \quad \widetilde{I}_{i}^{(*n)}\widetilde{I}_{i(n)}, \quad \widetilde{I}_{i}^{(\varepsilon * n)}\widetilde{I}_{i(n)}\right).
\end{gathered} \tag{1.15}$$

In Eqs. (1.15) and the second of (1.13) $\widetilde{M}^{(n)}$, $\widetilde{q}^{(n)}$, and $\widetilde{\eta}^{(n)}$ are functions of $\widetilde{e}_{ij}^{(*n)}$, $\widetilde{\varepsilon}_{ij(n)}$, $\widetilde{I}_{i}^{(*n)}$ $\widetilde{I}_{i(n)}$, $\widetilde{I}_{i}^{(\varepsilon * n)}$, $\widetilde{I}_{i(n)}$. Here strain anisotropy is not taken into account, and the existence of stress increment and magnetization potentials depending on n is assumed. The functions $\widetilde{M}^{(n)}$, $\widetilde{q}^{(n)}$, and $\widetilde{\eta}^{(n)}$ for the n-th loading are found from very simple experiments. The corresponding problems are solved by the method of successive approximations, analogous to the method in [10, 1] for the base and variable loads of isotropic bodies in the absence of an electromagnetic field. Not taking account of the change in the linearly elastic constants with the number of the loading for the theory of plasticity of anisotropic ferromagnets in the presence of a magnetic field, it is possible to use the analog of the generalized Rayleigh–Masing–Moskvitin principle [11, 12, 2, 1]

$$\begin{gathered}
\widetilde{M}^{(n)} = M^{(\ )}\left(\widetilde{e}_{ij}^{(*n)}\widetilde{\varepsilon}_{ij(n)}/\alpha_n^2, \quad \widetilde{I}_{i}^{(*n)}\widetilde{I}_{i(n)}/\beta_n^2, \quad \widetilde{I}_{i}^{(\varepsilon * n)}\widetilde{I}_{i(n)}/\alpha_n\beta_n^2\right), \\
\widetilde{q}^{(n)} = q\left(\widetilde{e}_{ij}^{(*n)}\widetilde{\varepsilon}_{ij(n)}/\alpha_n^2, \quad \widetilde{I}_{i(n)}^{(*n)}\widetilde{I}_{i(n)}/\beta_n^2, \quad \widetilde{I}_{i}^{(\varepsilon * n)}\widetilde{I}_{i(n)}/\alpha_n\beta_n^2\right), \\
\widetilde{\eta}^{(n)} = \eta\left(\widetilde{e}_{ij}^{(*n)}\widetilde{\varepsilon}_{ij(n)}/\alpha_n^2, \quad \widetilde{I}_{i(n)}^{(*n)}\widetilde{I}_{i(n)}/\beta_n^2, \quad \widetilde{I}_{i}^{(\varepsilon * n)}\widetilde{I}_{i(n)}/\alpha_n\beta_n^2\right),
\end{gathered} \tag{1.16}$$

where $M^{(\ )}$, q, and $\eta$ are functions corresponding to the base load, $\alpha_n$ is the parameter of the n-th mechanical loading, having the meaning of the coefficient of change of scale of the stress and strain axes and also of the axes of the increments of stresses and strains for the n-th loading (the parameters $\alpha_n$ are on the order of magnitude of 2), $\beta_n$ is the parameter of the n-th magnetic loading, having the meaning of the coefficient of change of scale of the axes of the fields and magnetization and also of the increments of the fields and magnetization for the n-th loading (the parameters $\beta_n$ are on the order of magnitude of 2); the $\gamma_n \equiv \beta_n \alpha_n^{1/2}$ are parameters characterizing the interaction of the magnetic and mechanical loadings. In the formation of material relations for the n-th loading principle (1.16) permits the expression of the corresponding functions in terms of the functions $M^{(\ )}$, q, and $\eta$ of the first loading.

When the values of quantities for the n − 1-th loading are known, and a model is fixed for the increments of the quantities, Eqs. (1.11) determine the quantities for the n-th loading.

A loading condition is a positive value of the energy dissipation function $\delta\chi/\delta t$

$$\delta\chi/\delta t > 0. \tag{1.17}$$

For the n-th loading the dissipation of energy $\chi = \chi^{(n)}$ is written as the sum

$$\chi = \chi^{(n)} \equiv \chi^{(n-1)} + \widetilde{W}^{(n)} - \widetilde{W}_e^{(n)}, \tag{1.18}$$

where $\chi^{(n-1)}$ is the energy dissipated up to the final instant of the $(n-1)$-th loading, $\widetilde{W}^{(n)}$, work for the increments of the quantities; $\widetilde{W}_e^{(n)}$, nondissipated part of the work for the increments of the quantities. For $\widetilde{W}_e^{(n)}$ we assume a model such that for a closed cycle the area of the hysteresis loop is equal to the energy dissipated in a cycle. Then

$$\widetilde{W}^{(n)} = \int\limits_{0,0,0}^{L,M,N} \left( \frac{\widetilde{\sigma}_i^{(n)}}{3\widetilde{\varepsilon}_i^{(*n)}}\, dL + \frac{\widetilde{H}^{(n)}}{2\widetilde{I}^{(*n)}}\, dM + \frac{\widetilde{H}^{(\varepsilon n)}}{2\widetilde{I}^{(\varepsilon *n)}}\, dN \right) + \frac{1}{2}\, \widetilde{\sigma}_{ij(n)}^{(\sigma)} \widetilde{\varepsilon}_{ij(n)}, \quad \widetilde{W}_e^{(n)} = \frac{1}{2}\, \widetilde{\sigma}_{ij(n)} \widetilde{\varepsilon}_{ij(n)} + \frac{1}{2}\, \widetilde{H}_{i(n)} \widetilde{I}_{i(n)}, \tag{1.19}$$

where

$$L = \widetilde{e}_{ij(n)}^{(*n)} \widetilde{\varepsilon}_{ij(n)}; \quad M = \widetilde{I}_i^{(*n)} \widetilde{I}_{i(n)}; \quad N = \widetilde{I}_i^{(\varepsilon *n)} \widetilde{I}_{i(n)};$$
$$\widetilde{\sigma}_{ij(n)}^{(\sigma)} = \frac{3}{2}\, [K_{pp\alpha\beta}\delta_{ij} + K_{ppij}\delta_{\alpha\beta}]\, \widetilde{\varepsilon}_{ij(n)}. \tag{1.20}$$

If the value of $\chi^{(n)}$ given by (1.18) – (1.20) does not satisfy inequality (1.17) with the use of model (1.11), Eqs. (1.5) and (1.6) in increments, and (1.13) – (1.15), this implies that there is an unloading, and the loading has the number n + 1. The model assumed is one possible variant. It is applicable when $S_{ij}^{[n]}$, $S_{ij}^{(In)}$, and $H_i^{(\varepsilon n)}$ are small in comparison with $S_{ij}^{(n)}$ and $H_i^{(In)}$, respectively, for simple and nearly simple loadings (see below). To construct other variants of the model it is necessary to give up the conditions of three-dimensional linearity, the existence of potentials, and the constancy of the orientations of the direction tensors and vectors.

2. For simple loading and strain [10] the direction stress and strain tensors and the direction magnetic field and magnetization vectors for any number of the loading n do not depend on the time t. A theorem of simple variable loading holds. Let us consider an incompressible anisotropic material characterized by the equalities

$$K = \infty, \ \mu_{ij\alpha\beta}^{[*]} = 0, \ \widetilde{\varepsilon}^{(*n)} = 0, \ 4\pi\, |I_{i(l)}| \gg |H_{i(l)}| \quad (l = 1, 2, \ldots, n),$$
$$\widetilde{\sigma}_i^{(n)} = 3\widetilde{\varepsilon}_i^{(*n)} \frac{\partial \Phi}{\partial \left( \widetilde{e}_{ij}^{(*n)} \widetilde{\varepsilon}_{(jin)} \right)}, \quad \widetilde{H}^{(n)} = 2I^{(*n)} \frac{\partial \Phi}{\partial \left( \widetilde{I}_i^{(*n)} \widetilde{I}_{i(n)} \right)},$$
$$\Phi = \sum_{r,s} A_{rs} \prod_{k=1}^{n} \left( \frac{2}{3}\, \widetilde{e}_{ij}^{(*k)} \widetilde{\varepsilon}_{ij(k)} \right)^{a_r^{(k)}} \prod_{l=1}^{n} \left( \widetilde{I}_i^{(*l)} \widetilde{I}_{i(l)} \right)^{b_s^{(l)}},$$
$$\sum_{k=1}^{n} a_r^{(k)} = (\gamma + 1)/2, \quad \sum_{l=1}^{n} b_s^{(l)} = (\delta + 1)/2, \quad \widetilde{\sigma}_i^{(In)} = 0, \quad \widetilde{H}^{(\varepsilon n)} = 0, \tag{2.1}$$

where the body forces and surface forces, the magnetic field, the magnetization, and the strains vary proportionally respectively to the parameters $\lambda_{(l)}(t)$, $\nu_{H(l)}(t)$, $\nu_{I(l)}(t)$, and $\mu_{(l)}(t)$, where $l$ is the number of the loading:

$$F_{i(l)} = F_i^0 \lambda_{(l)}(t), \quad T_{i(l)} = T_i^0 \lambda_{(l)}(t), \quad H_{i(l)} = H_i^0 \nu_{H(l)}(t),$$
$$I_{i(l)} = I_i^0 \nu_{I(l)}(t), \quad \varepsilon_{ij(l)} = \varepsilon_{ij}^0 \mu_{(l)}(t) \quad (l = 1, 2, \ldots, n), \tag{2.2}$$

where the quantities with superscript zero do not depend on the time. The loading and strain will be simple if

$$\lambda_{(l)}(t) = \nu_{H(l)}(t)\,\nu_{I(l)}(t) \quad (l = 1, 2, \ldots, n), \tag{2.3}$$

$$\lambda_{(n-1)} - \lambda_{(n)} = \frac{1}{(\gamma+1)A}\sum_{r,s} A_{rs} a_r^{(n)} (\mu_{(n-1)} - \mu_{(n)})^{-1} \prod_{k=1}^{n} |\mu_{(k-1)} - \mu_{(k)}|^{2a_r^{(k)}} \prod_{l=1}^{n} |\nu_{I(l-1)} - \nu_{I(l)}|^{2b_s^{(l)}},$$

$$\nu_{H(n-1)} - \nu_{H(n)} = \frac{1}{(\delta+1)A}\sum_{r,s} A_{rs} b_s^{(n)} (\nu_{I(n-1)} - \nu_{I(n)})^{-1} \prod_{k=1}^{n} |\mu_{(k-1)} - \mu_{(k)}|^{2a_r^{(k)}} \prod_{l=1}^{n} |\nu_{I(l-1)} - \nu_{I(l)}|^{2b_s^{(l)}}.$$

In Eqs. (2.1) and (2.3) $A_{rs}$, $a_r^{(k)}$, $b_s^{(l)}$, $\gamma$, $\delta$, A are constants. The stresses $\sigma_{ij(n)}$ are found in the form

$$\sigma_{ij(n)} = \sigma_{ij}^{0}\lambda_{(n)}(t). \tag{2.4}$$

It follows from (1.3) and the fourth of Eqs. (2.1) that

$$B_{i(n)} = B_i^0 \nu_{I(n)}(t). \tag{2.5}$$

The substitution of (2.4), (2.5), and (2.2) into (1.1), (1.2), and (1.4) gives the first of Eqs. (2.3). Maxwell's equations for magnetostatics and the boundary conditions are satisfied also. When (2.2) and (2.4) are satisfied the first two of Eqs. (1.3) and the fourth of (1.13) are satisfied, since the incompressibility conditions, the first three of Eqs. (2.1), are valid. To confirm this it is necessary to use the first, third, sixth, and eighth of Eqs. (1.11) and the second of (1.10). Since the potential $\Phi$ in (2.1) does not depend on the invariant N (cf. (1.19)), which is equivalent to satisfying the last two of Eqs. (2.1), the fifth and seventh of Eqs. (1.13) are trivially satisfied:

$$\widetilde{S}_{ij}^{0(In)} = 0, \quad \widetilde{H}_i^{0(\varepsilon n)} = 0.$$

The substitution of Eqs. (2.2) and the expressions for $\tilde{\sigma}_i^{(n)}$ and $\tilde{H}^{(n)}$ from (2.1) into the third and sixth of Eqs. (1.13) and taking account of the second and fifth of Eqs. (1.11) gives the second and third of Eqs. (2.3). Since conditions (2.2) and (2.4) give the constancy of the direction tensors and the conditions of the consistency theorem, the theorem is proved.

We note that for nonzero but small stresses $\tilde{S}_{ij}^{[n]}$ and $\tilde{S}_{ij}^{(In)}$ and the fields $\tilde{H}_i^{(\varepsilon n)}$ a loading close to (2.2) will be close to simple.

By using (1.16) it is possible to prove a theorem on variable loading. If the problem of the variable loading of a ferromagnet is solved by the method of successive approximations, at each stage it is necessary to solve the separate problem of magnetostatics for a known state of strain found from the preceding approximation, and the mechanical problem for known fields and magnetizations. We denote the k-th approximation by a superscript k in curly brackets. From equations of the type (1.11) the stresses $\sigma_{ij(n)}^{\{k\}}$, the strains $\varepsilon_{ij(n)}^{\{k\}}$, the magnetic field $H_{i(n)}^{\{k\}}$ are equal to the differences (n even) or sums (n odd):

$$\sigma_{ij(n)}^{(k)} = \sigma_{ij(n-1)}^{\{k\}} - (-1)^n \tilde{\sigma}_{ij(n)}^{\{k\}}, \quad \varepsilon_{ij(n)}^{\{k\}} = \varepsilon_{ij(n-1)}^{\{k\}} - (-1)^n \tilde{\varepsilon}_{ij(n)}^{\{k\}},$$
$$H_{i(n)}^{\{k\}} = H_{i(n-1)}^{\{k\}} - (-1)^n \tilde{H}_{i(n)}^{\{k\}}, \quad I_{i(n)}^{\{k\}} = I_{i(n-1)}^{\{k\}} - (-1)^n \tilde{I}_{i(n)}^{\{k\}}$$

of the corresponding quantities $\sigma_{ij(n-1)}^{\{k\}}$, $\varepsilon_{ij(n-1)}^{\{k\}}$, $H_{i(n-1)}^{\{k\}}$, and $I_{i(n-1)}^{\{k\}}$ existing before the beginning of the n-th loading, and certain fictitious quantities which result from solving the problem of the base loading under the action of the loads

$$(-1)^n \left[F_{i(n-1)} - F_{i(n)} + F_i\left(I_{(n-1)}^{\{k-1\}}, H_{(n-1)}^{\{k-1\}}\right) - F_i\left(I_{(n)}^{\{k-1\}}, H_{(n)}^{\{k-1\}}\right) - F_i\left(I_{(n-1)}^{\{k-1\}} - I_{(n)}^{\{k-1\}}, H_{(n-1)}^{\{k-1\}} - H_{(n)}^{\{k-1\}}\right)\right],$$

$$(-1)^n \left[T_{i(n-1)} - T_{i(n)} + T_i\left(H_{0(n-1)}^{\{k-1\}}\right) - T_i\left(H_{0(n)}^{\{k-1\}}\right) - n_j \sigma_{ij}\left(B_{(n-1)}^{\{k-1\}}, H_{(n-1)}^{\{k-1\}}\right) +$$

$$+ n_j\sigma_{ij}\left(B_{(n)}^{(k-1)}, H_{(n)}^{(k-1)}\right) + n_j\sigma_{ij}\left(B_{(n-1)}^{(k-1)} - B_{(n)}^{(k-1)}, H_{(n-1)}^{(k-1)} - H_{(n)}^{(k-1)}\right)\Big]$$

and the current density

$$(-1)^{(n)}(j_{i(n-1)} - j_{i(n)})$$

under the condition that in the material relations of the base load the scales of the axes of stresses and strains are changed by a factor $\alpha_n$, and the scales of the field and magnetization by a factor $\beta_n$. The notations used here appeared in Eqs. (1.1), (1.2), and (1.4).

3. Let us obtain the material relations for a polycrystalline steel sample. We consider first the base load. We choose $\sigma_{ij}$ and $H_i$ as determining parameters. Then the potential of the strains and magnetizations has the form

$$f = \varkappa_0(H, \sigma_i) H^2/2 - a_0(H, \sigma_i) H_i H_j \sigma_{ij} + \sigma^2/2K + \int_0^{\sigma_i} \varepsilon_i d\sigma_i, \tag{3.1}$$

where $\sigma_i$ and $\varepsilon_i$ are the intensities of the stresses and strains; $\varkappa_0$ and $a_0$ are functions of H and $\sigma_i$ which must be determined experimentally. Equation (3.1) yields the following expressions for the strains and magnetizations:

$$\varepsilon_{ij} = \frac{\partial f}{\partial \sigma_{ij}} \equiv \frac{1}{3K}\sigma\delta_{ij} + \frac{3}{2\sigma_i}\left(\varepsilon_i + \frac{\partial \varkappa_0}{\partial \sigma_i}H^2 - \frac{\partial a_0}{\partial \sigma_i}H_\alpha H_\beta \sigma_{\alpha\beta}\right)S_{ij} - a_0 H_i H_j, \tag{3.2}$$

$$S_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{\alpha\alpha}\delta_{ij},$$

$$I_i = \frac{\partial f}{\partial H_i} \equiv \left(\varkappa_0 + \frac{1}{2}\frac{\partial \varkappa_0}{\partial H}H - \frac{\partial a_0}{\partial H}\frac{H_\alpha H_\beta}{H}\right)H_i - 2a_0\sigma_{i\alpha}H_\alpha.$$

For the simple elongation of a long rod in the direction of the axis the second of Eqs. (3.2) can be rewritten in the form

$$I = \varkappa_0(H, \sigma_i) H + \frac{H^2}{2}\frac{\partial \varkappa_0(H, \sigma_i)}{\partial H} - 2a_0(H, \sigma_i)\sigma_i H - H^2\sigma_i\frac{\partial a_0(H, \sigma_i)}{\partial H}, \tag{3.3}$$

$$I = I_1,\ H = H_1, \sigma_i = \sigma_{11},$$

where $\sigma_{11}$ is the stress applied to the ends of the rod, and $H_i$ and $I_i$ are directed along the axis of the rod.

We approximate the experimental magnetization curves for a patented steel wire [13], choosing as an analytic relation the function given in [14];

$$\varkappa_0(H, \sigma_i)H + (H^2/2)\partial\varkappa_0/\partial H = \alpha_1 \operatorname{arctg}[\exp(\varkappa\sigma_i)\beta H], \tag{3.4}$$

$$2a_0(H, \sigma_i)H + H^2\partial a_0/\partial H = \alpha_2 \operatorname{arctg}[\exp(\varkappa\sigma_i)\beta H],$$

where $\alpha_1$, $\alpha_2$, $\varkappa$, and $\beta$ are experimental constants.

After integrating Eqs. (3.4), analytic expressions are obtained for the functions $\varkappa_0(H, \sigma_i)$ and $a_0(H, \sigma_i)$

$$\varkappa_0(H, \sigma_i) = \frac{2\alpha_1}{H}\operatorname{arctg}[\exp(\varkappa\sigma_i)\beta H] - \frac{\alpha_1 \ln\left[\exp(-2\varkappa\sigma_i) + \beta^2 H^2\right]}{\beta H^2 \exp(\varkappa\sigma_i)}, \tag{3.5}$$

$$a_0(H, \sigma_i) = \frac{\alpha_2}{H}\operatorname{arctg}[\exp(\varkappa\sigma_i)\beta H] - \frac{\alpha_2 \ln\left[\exp(-2\varkappa\sigma_i) + \beta^2 H^2\right]}{2\beta H^2 \exp(\varkappa\sigma_i)}.$$

Taking account of (3.4), Eq. (3.3) can be written in the form

$$I = (\alpha_1 - \alpha_2\sigma_i)\operatorname{arctg}[\exp(\varkappa\sigma_i)\beta H]. \tag{3.6}$$

The approximation of the experimental data in [14, 13] by Eq. (3.6) is quite satisfactory.

The constants $\alpha_1$ and $\beta$, are determined by comparing the experimental curves with Eq. (3.6) for $\sigma_i = 0$ [14]. After determining $\alpha_1$ and $\beta$, the constants $\varkappa$ and $\alpha_2$ are found for fixed $\sigma_i$ from the conditions

$$\varkappa = (1/\sigma_i)\ln(1/\beta H_\sigma),\ \alpha_1 - \alpha_2\sigma_i = (2/\kappa)I_*(\sigma_i),$$

where $I_*(\sigma_i)$ is the saturation magnetization, and $\sigma_i$ in parentheses indicates that the constants are calculated from the graph of I as a function of H for a given $\sigma_i$; $H_\sigma$ is taken from the I–H curve at a value of the magnetization equal to $(1/2)I_*(\sigma_i)$. The values of the constants for the patented steel wire [13] introduced into the model (3.2), (3.5) were determined from the experimental I–H curves for $\sigma_i = 0$ and $\sigma_i = 9.7 \cdot 10^9$ dyne/cm$^2$:

$$\alpha_1 = 950\ \mathrm{G},\ \beta = 7.8 \cdot 10^{-2}(1/\mathrm{Oe}), \alpha_2 = 6.5 \cdot 10^{-8}\ \mathrm{G \cdot cm^2/dyne},$$
$$\varkappa = 1.17 \cdot 10^{-11}\ \mathrm{cm^2/dyne}. \tag{3.7}$$

Equations (3.2) written for increments are valid for variable loadings. When the material relations for an isotropic body are written in this way principles analogous to (1.16) are valid in which for simplicity it is possible to take parameters of the change of scale equal to 2:

$$\tilde{\varkappa}_0^{(n)} = \varkappa_0(\tilde{H}^{(n)}/2, \tilde{\sigma}_i^{(n)}/2),$$
$$\tilde{a}_0^{(n)} = (1/2)\,a_0(\tilde{H}^{(n)}/2, \tilde{\sigma}_i^{(n)}/2),\ \ \tilde{\varepsilon}_i^{(n)} = 2\varepsilon_i(\tilde{\sigma}_i^{(n)}/2). \tag{3.8}$$

The functions $\varkappa_0$ and $a_0$ for steel are determined by Eqs. (3.5) and (3.7). At the given point small fields are considered which do not affect the form of the function $\varepsilon_i(\sigma_i)$. Therefore, function $\varepsilon_i(\sigma_i)$, which is known from experiment in the absence of a magnetic field, enters (3.8).

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# STRESS STATE OF A STRAIGHT ISOLATED CUT, LOADED FROM WITHOUT BY CONCENTRATED FORCES AND GROWING AT A CONSTANT RATE

E. N. Sher

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In [1] an investigation was made of the stress state of a straight isolated cut, developing at a constant rate under the conditions of antiplane deformation of the ideal theory of elasticity. Here, we consider cases of total self-similar loading and non-self-similar loading by concentrated forces, applied at the middle of the cut to its edges and depending arbitrarily on the time.

In the present work an analogous investigation is made within the framework of the ideal theory of elasticity for the case of plane deformation; here, use was made of results and ideas published in [2, 3].

In the unloaded elastic xy plane, at the initial moment of time t = 0, let a cut loaded by forces concentrated along its edges start to develop along the x axis from the origin of coordinates with the rate v. It is required to determine the stress state arising in the plane, specifically, the value of the coefficient of the field intensity of the stresses near the tip of the cut. The elastic displacements, as is well known [4], satisfy the following equations:

$$\begin{gathered} w_i = u_i + v_i, \quad \Delta u_i = \frac{1}{a^2}\frac{\partial^2 u_i}{\partial t^2}, \quad \Delta v_i = \frac{1}{b^2}\frac{\partial^2 v_i}{\partial t^2}, \\ \frac{\partial u_1}{\partial y} = \frac{\partial u_2}{\partial x}, \quad \frac{\partial v_1}{\partial x} = -\frac{\partial v_2}{\partial y}, \end{gathered} \tag{0.1}$$

where $u_i(x, y, t)$, $v_i(x, y, t)$ are the potential and solenoidal components of the displacement vector $w_i(x, y, t)$; $a$ and b are the velocities of the longidinal and transverse waves of the elastic plane.

The components of the stress tensors are expressed in terms of the displacements by the formulas

$$\begin{gathered} \sigma_{xx} = \mu\left[\frac{a^2}{b^2}\left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y}\right) - 2\frac{\partial w_2}{\partial y}\right], \\ \sigma_{yy} = \mu\left[\frac{a^2}{b^2}\left(\frac{\partial w_1}{\partial x} + \frac{\partial w_2}{\partial y}\right) - 2\frac{\partial w_1}{\partial x}\right], \quad \sigma_{xy} = \mu\left[\frac{\partial w_1}{\partial y} + \frac{\partial w_2}{\partial x}\right]. \end{gathered} \tag{0.2}$$

We consider the region of the upper half plane y > 0, bounded by the arc of the longitudinal wave, emitted at the initial moment of time (Fig. 1). In this region a solution of system (0.1) is sought, satisfying the following boundary conditions. At the edge of the cut, with $|x| < vt$, the external load is given: $\sigma_{yy} = -\sigma_y(x, t)$; $\sigma_{xy} = 0$. The form of the function $\sigma_y(x, t)$ will be refined in what follows.

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